Research on the Optimal Planting Scheme of Crops Based on Linear Programming Model and Simulated Annealing Model

Qi Chen *

School of Electromechanical Engineering, Guangdong University of Technology, Guangdong, China, 510006

* Corresponding Author Email: qi_chen_0208@163.com

Abstract. With the continuous growth of China's population, the contradiction between food security and the limited arable land resources has become increasingly prominent. Scientifically planning crop planting schemes and enhancing agricultural economic benefits have become key issues. Based on the data of farmland types, crop yield per mu, planting cost and sales unit price in rural areas in 2023, this paper constructs a linear programming model with the goal of maximizing profits for two scenarios where crop yield exceeds the expected sales volume (the excess part cannot be sold and the excess part is sold at half price). First, collect data such as crop type, cultivated land area, planting area, per-mu yield, sales unit price and planting cost, solve the linear programming model, and finally optimize the model results by using the simulated annealing algorithm. Research shows that. Under the two sales scenarios, there are significant differences in profits between the single-crop planting and mixed planting schemes of different plots, and the free value of the objective function drops from the initial 0.36 to 0.0077, effectively improving the accuracy of the model. This research provides a scientific basis for crop planting decisions from 2024 to 2030, contributing to an increase in agricultural income.

Keywords: Linear programming, simulated annealing model, crop planting optimization, profit maximization.

1. Introduction

With the increase in the world's population, the food issue has become a significant problem nowadays, and the cultivation of crops is facing unprecedented challenges. These challenges not only affect the selection of crop types and the decision-making on the number of crops to be planted, but also directly relate to the stability and development of society. How to grow efficiently on limited land to meet the growing food demands of mankind has become an urgent problem to be solved. To ensure the sustainable development of agricultural planting, this article must attach great importance to the protection of existing land. The balance of soil nutrients, as a key support for the growth of crops, the development and application of related technologies are particularly crucial. Because the same type of crops has the same demand for nutrients, when they are continuously planted in the same plot of land, it will lead to an imbalance of soil nutrients, resulting in a reduction in crop yields. Soil nutrients in farmland are important indicators for evaluating soil fertility, the foundation for crop growth, and a crucial guarantee for crop yield and quality [1]. Therefore, a crop planting plan is a necessary prerequisite for accurately managing soil nutrients, rationally planning inputs in agricultural production, protecting and improving the quality of cultivated land soil, and increasing farmers' and rural income [2].

Existing research has focused on exploring planting optimization. Zhang Zhansheng pointed out that planting optimization needs to integrate precise soil nutrient management, providing a replicable model for precise soil nutrient management [3]. Hu Wei studied the design of suitable cropping systems and proposed a coordinated optimization strategy for crop layout and fertilizer application in arid areas, providing model support for avoiding continuous cropping obstacles [4]. Mi Xiaotian utilized machine learning to establish for the first time a data-driven national standard for wheat nitrogen management, providing a quantifiable path for the optimization of all-domain planting [5]. However, the existing achievements are relatively few. The optimization models for the specific scenarios of yield and unexpected sales lack the combined application of linear programming and heuristic algorithms, making it difficult to meet the precise demands of actual planting decisions [6, 7].

In response to the above deficiencies, this paper takes the crop planting in rural areas from 2024 to 2030 as the research objective and combines the data of cultivated land and crops in 2023 to construct an optimization model suitable for the sales scenario of yield exceeding expectations, including the following goals. 1. Clearly define the optimal crop planting combinations for different sales scenarios. 2. Enhance the global optimization capability of the model through the simulated annealing algorithm. 3. Reveal the profit differences among various planting methods for different plots and provide practical decision-making solutions for agricultural production practices.

2. Materials and methods

2.1. Data acquisition and preprocessing

The research data are sourced from the open source website (https: //www.mcm.edu.cn/), mainly including types of cultivated land, categories of grown crops, cost control, unit price of sales, etc.

Considering that irrigated land, ordinary greenhouses and smart greenhouses can be planted for two seasons, and the sales volume, sales price and planting cost of the same crop on different farmlands are all different, in data processing, for the convenience of calculation, a year is divided into two quarters. The flat and dry land, terraced fields and sloping land that can only be planted for a single season are treated as normal planting in the first season and no planting in the second season. This article quantitatively presents the output distribution under different types of agricultural land and farming systems, which has certain research value in agricultural economics and land management. The distribution of cultivated land and the statistical results of plantable crops are shown in Figure 1.

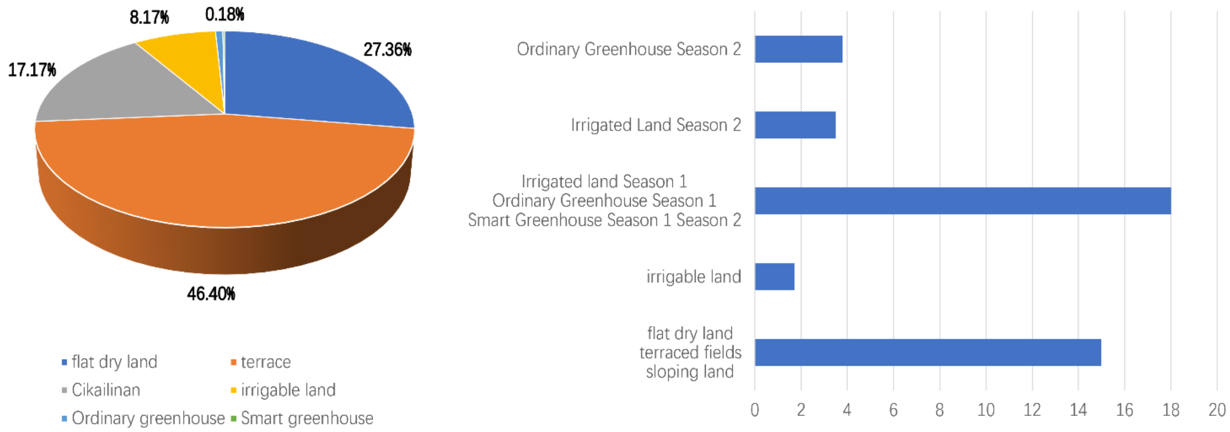

Figure 1. Statistics on the distribution of cultivated land and crops that can be grown.

2.2. Method Introduction

The method of collaborative optimization using linear programming and simulated annealing algorithms is adopted. First, establish a linear programming model with the goal of maximizing profits: for the scenario of unsold waste, directly discard the excess production part in the profit calculation; For the half-price sales scenario, the income from overproduction is counted at half price. The model strictly incorporates practical conditions such as plot area restrictions, crop rotation constraints (for instance, continuous planting of the same crop in a plot is prohibited), legume planting policies, planting density requirements, and cultivated land adaptability. Through the LINGO software, the planting plan for 54 plots ×41 crops ×14 quarters are efficiently solved [8].

To further enhance the quality of the scheme, the simulated annealing algorithm is introduced for global optimization. This algorithm takes the linear programming solution as the starting point, generates new solutions by randomly adjusting the crop combinations of some plots, and accepts short-term poor solutions through a probabilistic mechanism to avoid falling into local optimum. After 250 iterations of the temperature decay process, the algorithm significantly improved the initial solution and ultimately output the optimal planting plan that takes into account both constraints and profits, revealing the differentiated impact of single or mixed planting strategies on different plots on returns.

3. Model establishment and Solution

3.1. Situation One: Solving for the maximum benefit based on the assumption of waste in linear programming

3.1.1. Model establishment

Under the premise that both the objective function and the constraint conditions are linear and the objective is clear; this paper selects the linear programming model for solution.

3.1.2. Establishment of a single-factor linear programming model

(1) Formula for linear programming model

$$\max(min)\ z = c^T x_{ijt} \tag{1}$$

$$st\begin{cases}\sum_{j=1}^{n} a_{ij}x_{ijt} \le (=,\ge) b_{ij}, & i = 1,2,\ldots,m \\ x_{ijt} \ge 0, & j = 1,2,\ldots,n\end{cases} \tag{2}$$

Among them, c^T represents the parameters of the objective function, x_{ijt} represents the decision vector, b_{ij} represents the constant parameters of the constraint equation system, a_{ij} represents the coefficient parameters of the constraint equation system.

(2) Calculation of total sales

The formula for calculating the total sales of all crops:

$$Q = q \times s \tag{3}$$

Among them, Q represents total sales,q represents sales volume,s represents selling price.

(3) Total cost calculation

The formula for calculating the total cost of each crop:

$$C = x \times c \tag{4}$$

S represents the total cost, x is the quantity of crops planted, cis the planting cost.

(4) Establishment of the objective function

This model takes profit maximization as its optimization objective, and its mathematical expression is:

$$P = Q - C \tag{5}$$

$$P = \begin{cases}\sum_{j=1}^{41}\sum_{i=1}^{54}\left(q_{ij}^{A} \times s_{ij}^{A} - x_{ijt} \times c_{ij}^{A}\right), t = 1,3,5,7,\ldots,13 \\ \sum_{j=1}^{41}\sum_{i=1}^{54}\left(q_{ij}^{B} \times s_{ij}^{B} - x_{ijt} \times c_{ij}^{B}\right), t = 2,4,6,8,\ldots,14\end{cases} \tag{6}$$

Since irrigated land, ordinary greenhouses and smart greenhouses can be planted for two seasons, while flat land, terraced fields and sloping land can only be planted for A single season, for the convenience of calculation, a year is regarded as two quarters, that is, A (or t is odd) represents the first quarter of a certain year, and B (or t is negative) represents the second quarter of a certain year. For flat and dry land, terraced fields and sloping land that can only be planted in a single season, they should be treated as normal planting in the first season and no planting in the second season.

(5) Establishment of constraints

1) Constraint on planting area

To ensure that the number of mu of crops planted on each cultivated land does not exceed the total area of each cultivated land, it is necessary to impose restrictions on the planting area of each crop, namely:

$$\sum_{j=1} x_{ij} \leq a_i \tag{7}$$

2) Continuous cropping constraint

According to the laws of agricultural production, to ensure the balance of soil nutrients, the same crop cannot be continuously planted on the same piece of land, that is:

a) If the plot type is flat dry land, terraced fields and sloping land, then

$$x_t \times x_{t+2} = 0 \cdots, t = 1,2,3, \dots ,14 \tag{8}$$

b) If the plot type is irrigated land, ordinary greenhouse and smart greenhouse, then

$$x_t \times x_{t+1} = 0 \cdots, t = 1,2,3, \dots ,14 \tag{9}$$

3) Legume crop constraints

As the root fungi of leguminous plants fix nitrogen in the soil and restore the ecological environment, in order to protect the land, it is stipulated that leguminous crops should be planted on cultivated land at least once every three years starting from 2023, that is:

$$\sum_{t}^{t+3} x_{ij\cdot} \geq 0 \tag{10}$$

$$j = 1,2, \dots ,5; t = 1,2,3, \dots ,14 \tag{11}$$

j represents leguminous plants.

4) Output constraints

According to common sense analysis, the yield of a crop should be greater than or equal to its sales volume [9], that is:

$$\sum_{j=1}^{41}\sum_{i=1}^{54} x_{ij} \geq \sum_{j=1}^{41}\sum_{i=1}^{54} q_{ij} \tag{12}$$

5) Planting quantity constraint

Considering the rationality of cultivated land management, crop planting needs to have a certain density, neither too dense nor too loose. The reference literature data is as follows [10]:

$$\frac{x_{ijt}}{a_j} \in \{0.002,0.400\} \tag{13}$$

6) Constraints on the types of cultivated land for crop cultivation

Because different crops have different requirements for the type of cultivated land, for instance, soybeans cannot be grown in irrigated land, that is

$$j \in [1,16], i \in [1.26] \tag{14}$$

$$j \in (16,35.36,37), i \in [27,34] \tag{15}$$

$$j \in [17,34], i \in [27.54] \tag{16}$$

$$j \in [38,41], i \in [51,54] \tag{17}$$

3.1.3. Optimize the model integration presentation

$$P = Q - C \tag{18}$$

$$P = \begin{cases} \sum_{j=1}^{41}\sum_{i=1}^{54}\left(q_{ij}^{A} \times s_{ij}^{A} - x_{ijt} \times c_{ij}^{A}\right), t = 1,3,5,7,\dots,13 \\ \sum_{j=1}^{41}\sum_{i=1}^{54}\left(q_{ij}^{B} \times s_{ij}^{B} - x_{ijt} \times c_{ij}^{B}\right), t = 2,4,6,8,\dots,14 \end{cases} \tag{19}$$

$$st. \begin{cases} \sum_{j=1} x_{ij} \le a_i \\ x_t \times x_{t+2} = 0, t = 1,2,3,\dots,14 \\ x_t \times x_{t+1} = 0, t = 1,2,3,\dots,14 \\ \sum_{t}^{t+3} x_{ij} \ge 0, j = 1,2,\dots,5; t = 1,2,3,\dots,14 \\ \sum_{j=1}^{41}\sum_{i=1}^{54} x_{ij} \ge \sum_{j=1}^{41}\sum_{i=1}^{54} q_{ij} \\ \frac{x_{ijt}}{a_j} \in \{0.002, 0.400\} \\ j \in [1,16], i \in [1.26] \\ j \in (16,35.36,37), i \in [27,34] \\ j \in [17,34], i \in [27.54] \\ j \in [38,41], i \in [51,54] \end{cases} \tag{20}$$

3.2. Situation Two: Solving for the maximum profit of selling at half price based on the assumption of linear programming

Modification of the single-factor linear programming model

(1) Calculation of the total income exceeding the portion

Calculation formula for selling the excess part of each crop at half price:

$$Z = \frac{1}{2} \times s \times (x - q) \tag{21}$$

Among them, Z represents the total income of the excess part, s represents the selling price, x represents the quantity of crops grown, and q represents the sales volume.

(2) Establishment of the objective function

This model takes profit maximization as its optimization objective, and its mathematical expression is:

$$P = Q - C + Z \tag{22}$$

$$P \begin{cases} \sum_{j=1}^{41}\sum_{i=1}^{54}\left[q_{ij}^{A} \times s_{ij}^{A} - x_{ijt} \times c_{ij}^{A} + \frac{1}{2} \times s_{ij}^{A} \times \left(x_{ijt} - q_{ij}^{A}\right)\right], t = 1,3,5,7,\dots,13 \\ \sum_{j=1}^{41}\sum_{i=1}^{54}\left[q_{ij}^{B} \times s_{ij}^{B} - x_{ijt} \times c_{ij}^{B} + \frac{1}{2} \times s_{ij}^{B} \times \left(x_{ijt} - q_{ij}^{B}\right)\right], t = 2,4,6,8,\dots,14 \end{cases} \tag{23}$$

A represents the first section and B represents the second quarter. The reasons and handling methods are the same as above.

3.3. Improvement of the simulated annealing model

3.3.1. Introduction to the simulated annealing model

The simulated annealing model is a type of heuristic algorithm. A heuristic algorithm utilizes the information obtained in the original search process during the search for the optimal solution, and this information will improve the search process.

The simulated annealing algorithm is an optimization method established based on the metal annealing mechanism. It can find the global maximum and minimum points of the objective function in a probabilistic sense by using random search technology. The simulated annealing algorithm analogues this process to an optimization problem. The state space corresponds to the solution space of the problem, and energy corresponds to the value of the objective function. The algorithm starts with a random initial solution and undergoes a certain number of random perturbations at each temperature to generate new solutions. If the new solution is better than the current one, accept the new solution as the current one. If the new solution is worse than the current one, the new solution will be accepted with a certain probability. This probability gradually decreases as the temperature drops, enabling the algorithm to conduct a wide range of searches at high temperatures and gradually converges to the vicinity of the local optimal solution at low temperatures.

3.3.2. Metropolis rule

At a certain temperature in the annealing model, the current state is a, and its corresponding objective function value is E(a). After random perturbation, the new state is b, and its corresponding objective function is E(b). For the study of minimum value problems, if the objective function of the new state is superior to the current state function, that is, E(b)>E(a), it can be considered that the new state is unconditionally accepted, that is, the current state is updated to the new state. If the objective function of the new state is not as good as the current state, that is, E(b)<E(a), then the probability of accepting this state under certain conditions is

$$P = \exp\left(-\frac{E(b) - E(a)}{T}\right) \tag{24}$$

Here, T represents the current temperature.

3.3.3. Simulation process

This time, the built-in simulated annealing algorithm toolbox of matlab is adopted to handle the problem. In this paper, the initial value of the objective function is defined as 0, and the upper and lower limits are 0 and 1 respectively. The set stopping criterion is: 250 iterations, with a target value of 0.007. The initial temperature was 100 degrees. To visualize the annealing process, this paper uses the Plot functions function to display the minimum value of the function after each iteration and its corresponding value of x; The change of the termination condition of the iterative function; Draw the changes in temperature; The value of x after each iteration; The objective function corresponding to each iteration.

3.3.4. Analysis of Simulation Results

Best function value: The minimum value of the objective function found after each iteration can be seen from the figure. At the beginning, the model selected a local optimal solution of about 0.36, but as the model progresses, this optimal solution gradually decreases until the final optimal solution is 0.0077.

Best point plots: The value of x corresponding to the minimum value of the objective function after each iteration is 8e-4.

Stopping criteria: Draw the variation of the termination condition of the function. Since only one termination condition is set here, there is only one graph here.

Temperature plot: Plot the changes in temperature. The initial set temperature is 100, and it gradually decreases as the iteration progresses.

Current point: The value of x corresponding to the end of each iteration. As can be seen from the figure, it is close to 7e-4 at the end.

Current function value: The objective function after each iteration can be seen from the graph. At the beginning, the data distribution is rather scattered, but after each iteration, the data gradually approaches.

The results of the simulated annealing algorithm are shown in Figure 2:

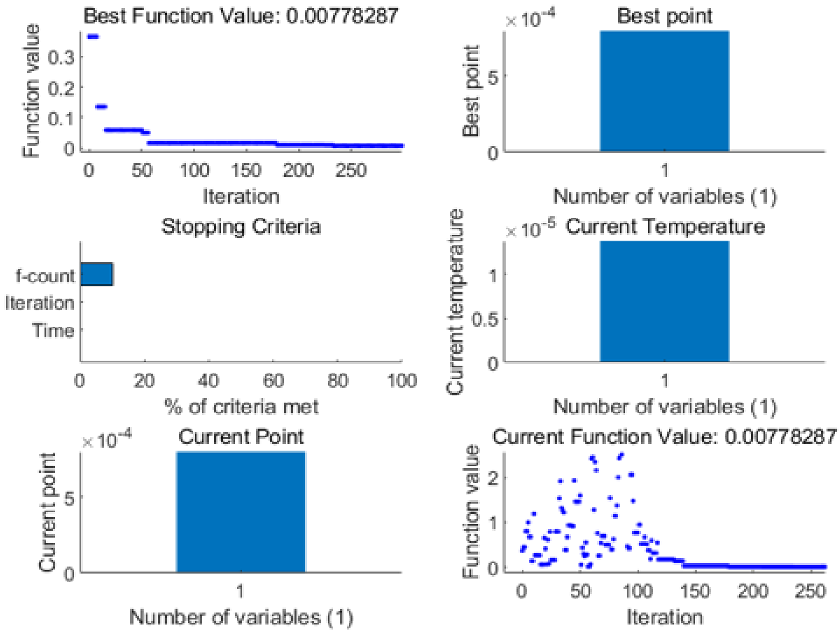

Figure 2. The results of the simulated annealing algorithm.

Detailed data of the simulated annealing iterative process are shown in Table 1

Table 1. Simulate the iterative process of annealing.

Iteration	f-count	Best f(x)	Mean f(x)	Temperature
0	1	0.366939	0.366939	100.000000
10	11	0.135387	1.186730	56.880000
20	21	0.058522	0.411975	34.056200
30	31	0.058522	0.788349	20.390700
40	41	0.058522	0.935402	12.208700
50	51	0.058522	1.600150	7.309770
60	61	0.015961	2.420770	4.376630
70	71	0.015961	0.690473	2.620450
80	81	0.015961	1.139620	1.568960
90	91	0.015961	2.580270	0.939395
100	101	0.015961	0.763653	0.562450
110	111	0.015961	0.378654	0.336760
120	121	0.015961	0.172984	0.201631
130	131	0.015961	0.172984	0.120724
140	141	0.015961	0.029394	0.072282
150	151	0.015961	0.029394	0.043278
160	161	0.015961	0.029394	0.025912
170	171	0.015961	0.029394	0.015515
180	181	0.010365	0.010365	0.009289
190	191	0.010365	0.010365	0.005562
200	201	0.010365	0.010365	0.003330
210	211	0.010365	0.010365	0.001994
220	221	0.010365	0.010365	0.001194
230	231	0.010021	0.010021	0.000715
240	241	0.007783	0.007783	0.000428
250	251	0.007783	0.007783	0.000256
260	261	0.007783	0.007783	0.000153
270	271	0.007783	0.007783	0.000092
280	281	0.007783	0.007783	0.000055
290	291	0.007783	0.007783	0.000033
300	301	0.007783	0.007783	0.000020

3.4. Interpretation of result

As can be seen from the above, most of the plots are single-crop plantations, but there are still some plots that are mixed plantations. It can be observed that there are significant differences in profits between monoculture and combination cropping for different plots. It cannot be simply assumed that monoculture or combination cropping on a certain plot can maximize profits.

4. Conclusion

This study constructed a linear programming model and two types of scenarios that effectively adapt production to sales exceeding expectations. By clearly defining the constraint conditions and objective functions to provide precise crop planting combination suggestions for different plots, the simulated annealing algorithm significantly improves the optimization accuracy of the model, converging the optimal value of the objective function to 0.0077 and reducing the risk of profit fluctuations. Meanwhile, this article reveals the optimal planting methods for the mainstream plots of terraced arid land, providing quantitative basis for rural agricultural production decisions from 2024 to 2030 and contributing to the increase of agricultural income. Future research can incorporate meteorological data and dynamic monitoring data of soil nutrients to enhance the current ability to imagine environmental changes and extend the model to multi-regional and multi-crop systems. Enhance the intelligent level of agricultural production.

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